The Real World
as a
Hologram

The dS/dS correspondence

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hep-th/0407125 + in progress

DBT in the Sky
M. A., D.Ti, E.S.

CF A. Karch,
Auto localization
E.S., M. Fabinger
D-Sitter+
Intersection

CF L. Kofman, A. Linde,
X. Liu, A. Maloney,
L. McAllister, E.S.,
Moduli Trapping
The dark energy is currently the largest component of the universe. \( \Lambda \geq 0 \) and the observations of CMBR are two of the most striking sets of recent data. I want to understand "the nature of the dark energy" and the microphysics of inflation.
These are usually discussed via simple effective field theory models at 2 derivative level
\[ Z = 6 \varepsilon I_{ij} \delta x^i \delta x^j - V(\phi) \]

e.g. • "What is the equation of state of the dark energy?"
• "How do we obtain a sufficiently flat slow-roll potential?"

But Gibbons/Hawking calculation of the dS entropy \( S = \frac{A}{46^n} \)
suggests there is a lot more physics in dS than this.
This talk:

1. Holographic duality for the dS static patch

2. Applications, including

QFT interpretation of the fact that it takes forever for a brane probe to reach a horizon \( \rightarrow \) new mechanism for inflation, with distinctive observational prediction (non-Gaussianity)
dS(d) static patch (spatial)

\[ g_{\alpha\beta} = 0 \quad \Theta_{\alpha\beta} = 1 \quad g_{\alpha\beta} = 0 \]

CFT on dS(d-1)

\[ \Theta_{s} \]

localized graviton

\[ E = 0 \quad w = 0 \]

\[ E = \frac{1}{R} \quad w = \frac{2}{\pi} R \]

\[ ds^2 = \sin^2 \frac{w}{R} ds^2_{dS_{d-1}} + dw^2 \]

\[ ds_{dS_d} \]

\[ E = 0 \quad w = \pi R \]

\[ \text{would be } \sinh \frac{w}{k} \quad \text{for AdS}_d \]

\[ dS_{d-1} \]
For $w \ll R$, $|w-\pi R| \ll R$ the metric for $\text{AdS}_d$ with $dS_{d-1}$ slices is the same as that for $dS_d$ with $dS_{d-1}$ slices.

$\Rightarrow$ both equivalent to CFT on $dS_{d-1}$ for energies $0 < E \ll \frac{1}{R}$
In this region, the (d-1)-dim'l theory exhibits a mass gap

\[ M^2_\text{CFT on dS}_{d-1} \geq \frac{(d-2)^2}{4 \, R^2} \]

\text{(Karch)}

This is \underline{not} an energy gap

\[ E = \sqrt{g_{00}} \, E_{\text{proper}} \rightarrow 0 \text{ at horizon} \] (mass "warped down" at horizon)
At higher energies $E \to \frac{1}{R}$, $AdS_d + dS_d$ differ:

In $dS_d$, $g_{00}$ reaches a maximum ($\alpha$ then goes down again on the other side).

- There is a $(d-1)$-dimensional graviton mode with a finite planck mass (Karch).

\[ M_{d-3}^{d-1} \sim R M_{d-1}^{d-2} \sim (R M_d)^{(d-2)/(d-3)} \]

induced by the CFT.
The dS/dS Correspondence:

 Altogether: dSd with radius R is holographically equivalent to 2 CFTs with entropy \( S \sim (R^d \text{vol})^{d-3} \) on dS_{d-1} of radius R cut off at \( E \leq \frac{1}{R} \) and coupled to each other and to \((d-1)\)-dimensional gravity.
Similar to

2-Throated RS (Warped Compactification) \( \rightarrow \) \( ds_{d-1} = 4 \)

\[ g_{00} = 1 \]

\[ g_{00} \ll 1 \]

QFT

\( E = \sqrt{g_{00}} M_{UV} \ll M_{UV} \)

localized graviton

\( E \sim M_{UV} \)

Cf. Gubser, Herms V, 
Hawking, Maldacena, Strominger 
CKP, KKLT, DKKLS
Remarks:

- The above derivation is similar to that leading to AdS/CFT (2 descriptions of same low energy regime).

- So far, both sides have gravity (like in RS/warped compactifications). Still useful since for large $r$ the bulk of the entropy is carried by the CFT.

- Can go further:

\[ dS_2 \rightarrow \text{CFT on } dS_{d-1} \rightarrow \cdots \text{ QM.} \]

in progress

\[ \text{in progress} \]
Like in AdS/CFT, this general dictionary will have many different microphysical realizations (supercritical). Some microphysical results e.g.

Going fully out on Coulomb branch manifests the entropy on branes:

\[
\text{IR} \quad \text{UV} \quad \text{AdS} \quad \text{IR} \quad \text{UV (cut off)} \quad \text{AdS}
\]
\[ S_{\text{max}}^{\text{KKLT}} \sim \frac{1}{N_{\text{vac}}} \left( \frac{\mathcal{S}_{\text{H}}}{b_3} \right)^{b_3} \]

the number of multifundamental states (microscopically string junctions ending on \( -b_3 \) bunches of branes) one can obtain on the Coulomb branch.

ES Sep '03

- In un-tuned \((K = l_s)\) examples one finds \[ S \sim \varphi^2 \]

correspondence point accounted for by unbound strings

M. Fabinger & E.S. Sperly '03 "D-Sitter"
Going only partially out on the Coulomb branch: brane probes of the bulk

\[ \sin(h) \frac{w}{\rho} = \left( \frac{P}{R} \right)^2 \]

\[ S_{DBI} = -\sigma_B \int d^{d-1}y \sqrt{g} \left( \frac{P}{R} \right)^{d-1} \sqrt{1 + \frac{R^4}{\rho^2} g^{\beta\beta} \frac{dq^2}{\rho^2 + \rho^2}} \]

encodes higher derivative field theory action, thermally averaged over bulk CFT d.o.f. more later...
One of the main questions to ask with a holographic duality is the set of observables.

E.g. $\text{AdS/CFT}$ $\text{CFT}$ Green's functions

In our codimension 1 $\text{dS/dS}$, as in $(\text{RS} = \text{CFT} + \text{gravity})$, the presence of $d-1$ gravity yields a residual puzzle (bulk of throat is large-$S$ field theory, but cutoff physics not precisely defined.)
Once we are down to

\((d=3) \rightarrow (d=2) \rightarrow (d=1)\)

gravity is no longer dynamical

\(\rightarrow\) should get a precise definition

Gravity does persist in the constraints. e.g. the Hamiltonian

constraint imposing time - reparameterization invariance

has interesting implications
Quantum world line of a relativistic particle

\[ S = m \int dt \, N \left( \frac{\dot{\theta}^2}{N^2} - 1 \right) \]

\[ N = \sqrt{g_{00}} \]

Hamiltonian constraint \[ \rightarrow \]

\[ S = m \int dt \, \sqrt{1 - \dot{\theta}^2} \]

\[ \Rightarrow \text{speed limit } \dot{\theta} < 1 \]

In our case, the role of \[ \dot{\theta}^2 \]

is played by \[ L_{c.a.m.} + L_{com} \]
The speed limit is then a bound on energy scale

\[ L = \sum_{\text{can}} E < C_1, \]

the same cutoff scale we had before.

This kind of t-reparameterization invariance Quantum Mechanics has been used by many (e.g. Hartle) as a toy model of quantum gravity. Here, it is the real thing!
- Thermal equilibration timescale \(<\) decay or Poincare recurrence timescale

- Decays mediated by vacuum bubbles also look like vacuum bubbles in the dual low energy \(\mathcal{FT}\)

\begin{center}
\begin{tikzpicture}
\node at (0,0) {\(\mathcal{D}_s\)};
\draw[thick,blue] (0,-3) -- (0,3);
\draw[thick,orange] (0,-2) -- (0,2);
\draw[thick,black] (0,-1) -- (0,1);
\node at (-3,0) {CFT on \text{dS}(d-1)};
\node at (3,0) {CFT on \text{dS}(d-1)};
\end{tikzpicture}
\end{center}
Now let me consider some of the physics of brane probes: (approximate) Coulomb branch of the dual.

In GR, it takes forever for a probe to reach a horizon in static coords

\[ ds^2 = -dt^2 \frac{1}{h} + \frac{dr^2}{h} + \cdots \]
This is holographically dual to:

A rolling scalar field $\alpha$ coupled to other fields $X_i$ via e.g. $g_i^2 \lesssim \alpha^2 X_i^2$ coupling.

Experiences large back reaction on its motion as $\alpha \to 0$ due to quantum effects of $X_i$.

$\hspace{1cm}\nabla_\alpha$
In SUSY theories, light states appear where symmetry enhanced symmetry points arise at complex codimension 1, 2, 3, ... (in SUSY theories, these points arise where light states contribute to the effective action, virtual particle production creates scalar field "moduli")

\[ \mathcal{L} = \sum_{i=1}^{3} \phi_i \]

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As we approach the origin, the production of $X$ particles is controlled by
\[
\frac{\dot{\omega}}{\omega^2} \rightarrow \frac{\dot{m}_X}{m_X^2} = \frac{\alpha}{g e^2}
\]
dominates at weak coupling.

One also has corrections to the effective action; e.g.,
\[
N = 4 \text{ SUSY} + \ldots
\]
\[
\gamma \propto \alpha^2 \left( 1 + N \frac{\alpha^2}{\alpha^4} + \ldots \right)
\]
dominates at strong coupling.
Weak coupling:  

\[ N_k = e^{\frac{-\pi (k^2 + g^2 \mu^2)}{g^2}} \]

Moduli trapping:  

\[ \Rightarrow N_X = \int \frac{d^3 k}{(2\pi)^3} N_k = \left( \frac{g^3}{2\pi} \right) e^{-\frac{\pi g^2 \mu^2}{g^2}} \]

\[ \Rightarrow \text{energy density} \]

\[ \rho_X = \int \frac{d^3 k}{(2\pi)^3} N_k \sqrt{k^2 + g^2 (\kappa(\eta))^2} \]

\[ \geq N_X \sqrt{\epsilon(\eta)} \]

As \( \epsilon \) rolls past the ESP, the masses of the created \( X \) particles grow \( \sim \sqrt{\eta} \)  

\[ \Rightarrow \text{effective potential traps} \epsilon \text{ near ESP} \]

\[ \frac{\dot{\epsilon}}{\epsilon^2} \ll 1 \]

\[ \frac{\dot{\epsilon}}{\epsilon^2} > 1 \]

\[ \frac{\dot{\omega}}{\omega^2} \ll 1 \]
We studied this numerically to account for the full back reaction on $\Omega$.

- With gravity, Hubble expansion →
  - Hubble friction limits scanning range of $\Omega$ (e.g., kinetic, $p_v$, etc.)
  - Hubble expansion dilutes particles that are produced
  - Near-ESP trapping is enhanced by Hubble friction

Applications:
- Vacuum Selection
- Trapped inflation
In the case of strong 't Hooft coupling $\lambda \gg 1$, for 4d CFTs (e.g. $N=4$ SYM) one can use the gravity side of AdS/CFT to find

\[ N \frac{\dot{\alpha}^2}{\alpha^4} < 1 \]

(Change notation: $\mathcal{F} = \mathcal{F}(n, \alpha)$)

$c$ = stretched string $m_c = \mathcal{F}$ in static configuration

Motion on Coulomb branch toward origin is motion of explicit D3-brane toward Poincare patch horizon

\[ \text{Dramatic Slowdown as approach } \alpha = 0 \text{ ESP "D-deceleration" relative to 2-derivative action (including exact metric in which } \alpha = 0 \text{ is at finite distance on moduli.)} \]
$S = \int d^4x \sqrt{-g} \frac{1}{2} \partial \phi \partial \phi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + J \phi$

DBI for brane, potential generated

Compactification, the action for $\mathcal{g}$ is coupled to 4d gravity (via Calabi-Yau)

Karch, Kehrein, McAllister, Mandal, Trivedi

K"all"en, K"all"en, N"ardec

Chern洱

(Chen rescaled $\sqrt{g} \to \sqrt{\rho}$)
We will be particularly interested in the case with \( V(\Phi) = m^2 \Phi^2 \).

In the presence of such a term, the AdS throat is generically closed off at a scale \( \Phi \approx \sqrt{\frac{m}{G_s}} \), which can be understood via

\[
\Delta M_x \text{ induced from } M_{5}\]

back reaction on throat metric from brane energy \( m^2 \frac{e^2}{x} \).

Also checked for self-consistency against particle production, acceleration, ... the model will be fine-tuned but consistent and predictive.
This leads to equations of motion for $\Phi$ and the metric scale factor ($ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$; $f(\Phi) = \frac{1}{\Phi}$ for AdS)

\[ \ddot{\Phi} + \frac{3f'}{2f} \Phi^2 - \frac{f'}{f^2} + \frac{3H_1}{f^2} \dot{\Phi} + (V + \frac{f'}{f^2})y_3 = 0 \]

Solution

\[ \ddot{\Phi} + \Phi^2 - \frac{\sqrt{\Phi}}{t} + \cdots \]

$y_3 \gg 1$ for late times

Potential force from large $y$

The potential is suppressed

$3H^2 = \frac{1}{g_5M_p^2} p = \frac{3}{g_5M_p^2} \left[ \frac{\sqrt{y}}{f} + (V - f'') \right]$
Solving for the dynamics, one finds a variety of interesting cosmological phases, including inflation on a steep potential

new mechanism for slow roll inflation

\[ V = m^2 \phi^2 \]

\[ a(t) = a_0 + \frac{t}{\dot{\phi}} \]

\[ N_e \sim \frac{1}{3} \log \left( \frac{\mathcal{P}_{\text{start}}}{\mathcal{P}_{\text{end}}} \right) \]

\[ \sqrt{\frac{\lambda}{\phi^4}} \lesssim 1 \]

\[ \Rightarrow \text{correct to integrate out the } \chi \text{s.} \]
Now let us calculate the density perturbations in this inflationary phase. Before delving into the details, a few main points that can be seen from the structure of our DBI action:

\[ L = \int d^4 x \sqrt{g} \left( \frac{\mathcal{F}^2}{2} \right)^4 \sqrt{1 - \frac{1}{\mathcal{F}^2} \left[ \mathcal{F}^2 \mathcal{G}^2 - \frac{1}{6} \mathcal{F}^2 \mathcal{G}^2 \right]} \]

make perturbations (for illustration)

1. Gaussian perturbations travel at a low sound speed \( c_s = \frac{k}{f} \):

\[ \mathcal{L} = \frac{\partial^2}{\partial t^2} \mathcal{G} - \frac{k^2}{a^2} \mathcal{G} + \cdots \]

2. Derivatives of \( \sqrt{1 - \mathcal{G}^2} \) \( \frac{1}{(1 - \mathcal{G}^2)^{3/2}} \)

will lead to freeze-out of modes at "sound horizon" \( c_s/H \ll H^{-1} \)
(2) Non-Gaussian Corrections (interactions) come with powers of $\gamma$, and hence are enhanced in our deceleration ($\gamma \gg 1$) phase.

Again, more derivatives w.r.t fluctuation $\gamma \Rightarrow$ more powers of $\frac{1}{\sqrt{1 - \nu^2}} = \gamma$

which is large in the background

Specifically

\[ L_3 = a^3 \gamma^3 \frac{\dot{a}}{a^4} \gamma^5 \]

\[ \frac{L_3}{L_2} = \gamma^2 \gamma \frac{\dot{a}}{2a^4} \]

So

will lead to a lower bound on non-Gaussian contribution to power spectrum, falsifiable prediction.
Scalar power spectrum

\[ P_k^3 = \frac{1}{4\pi^2} \left| C_+ - C_- \right|^2 \frac{g_s}{\mathcal{V}^4} \]

\[ n_s - 1 = \mathcal{O}(\mathcal{E}^2) + \text{(tilt from } C_+ - C_-) \]

\[ r = \frac{p}{\rho s} = \frac{16 \mathcal{E}}{y} \]

Non-Gaussianity

\[ \frac{Q_{111}}{Q_{22}} \sim \frac{y^2}{H^2} = \left( \frac{y^2 \langle P_k^3 \rangle}{\mathcal{V}} \right)^{1/2} \]

\[ \mathcal{K}_{k_1 k_2 k_3} = -\frac{i g_s^2}{2 \mathcal{V}} \int d^3(\Sigma_k \vec{E}_k) \frac{H^6}{\mathcal{V}^3} \left[ \frac{1}{k_1^2 k_2^2 k_3^2} \right] \]

\[ \int_0^\infty dr \frac{H^2}{2 \mathcal{V}} e^{-\frac{i \Sigma_k \vec{E}_k \cdot \vec{r}}{3}} \left( \frac{k_1^2 k_2^2 k_3^2}{y^2} \right) + \text{c.c.} \]

\[ \mathcal{K}_{k_1 k_2 k_3} = \mathcal{O}(\mathcal{E}^2) + \text{(tilt from } C_+ - C_-) \]

\[ \gamma \leq 80 \]

\[ \text{Need to check data for this 3-point function} \]
Tuning:

- \( \lambda \leftrightarrow \left(\frac{R}{L_s}\right)^2 \approx 100 - 1000 \)

required to suppress scalar power spectrum

- \( \lambda \neq g_s N \) in general
  - e.g. orbifold \( (\mathbb{Z}_2)^k \rightarrow \lambda = g_s N l^k \)

- Non linear effects grow with time \( \rightarrow \)
  - we are forced to \( \Phi \) start \( \sim M_p \)
  - (on verge of functional fine tune)

- Anyway, the model is Predictive \( \rightarrow \) let Data decide!
Remarks:

• Inflation $\nRightarrow$ Gaussian perturbations

Conversely, if still Gaussian after WMAP, Planck it is a real test of traditional inflation models

* Interactions that help slow the inflaton can lead to significant N.G.

• This new mechanism & predictions came from worrying about holography and duals of horizon physics – "formal" stuff
Much to do:

\(\text{(dS/dS)}\)

- Observables in 2d, 1d duals of dS
cf. 2d cosmology (Handle "spacetime & n.")

- Symmetries (e.g. bulk diffGeo; So(d-1) x transl.) in dual description

- Inflationary \(\rightarrow\) us \(\rightarrow\) c.c.

Spacetimes (cf. Myers "tall ds")

- Does dS/dS help us understand initial conditions, probability for inflation
  - cutoff changes with time (cf. RG flow): not just "box of gas"

- Black hole excitations & entropy
- Generalize to late-time decays
- **DBI**

- Data analysis for our specific 3-point function (cf. Adams, Creminelli, Baldaia)

- Space of models

  - e.g. not just warped throats; Hamiltonian constraint \( \rightarrow \) speed limit on moduli space occurs more generally.