Supersymmetry predicts:

- Many new particles to discover at LHC*
- A light Higgs†
- An era of perturbative control‡

I will discuss Strategy and Tactics for precise computation of physical masses in terms of Lagrangian parameters, and results for the MSSM squarks and Higgs.

*most likely
†probably
‡hopefully
Masses are key observables in SUSY.

Most of what we do not already know about Supersymmetric extensions of the Standard Model involves the soft breaking terms with positive mass dimension.

Predictions of specific models (Minimal Supergravity, Gauge Mediation, Anomaly Mediation, Extra-dimensional Mediation, ...) allow/require precise calculations.

The apparent unification of gauge couplings in the MSSM invites us to extrapolate the soft masses up to high scales, to see if they obey some organizing principle.
A case study (Snowmass 2001 P3 working group):

RH sleptons, LH sleptons, LH squarks

Assumptions:
2% uncertainty in $M_{\text{gluino}}$
1% uncertainty in $M_{\text{squarks}}$
0.5% uncertainty in $M_{\text{sleptons}}$
1% uncertainty in $\alpha_s$
0% theoretical uncertainty (!)

Goal: make the last, unreasonable, assumption as close to reasonable as possible.

(See also: Allanach, Blair, Kraml, Martyn, Polesello, Porod, Zerwas, hep-ph/0403133, and previous.)
TeV-scale SUSY will provide an interesting laboratory for quantum field theory:

- Fundamental scalar particles
- No new dimensionless couplings in Lagrangian
- QCD coupling is perturbative at the TeV scale, but still strong enough to require multi-loop calculations

Corrections to Higgs masses at 1 loop go like $y_t^2$, at 2 loops go like $y_t^2 g_3^2$.

 Corrections to gluino mass involve $C g_3^2$ with $C = 3$, rather than $C = 4/3$ for quarks. So, the QCD coupling for the gluino is effectively $9/4$ larger.

Corrections to squark, quark masses get large effects from the strongly-coupled, heavy gluino.

**Two-loop corrections to observables will be mandatory if SUSY is correct**
Another key feature of the problem: many distinct particles.

- **2-loop diagrams involve many different mass scales simultaneously.**
  
  For example:

  \[ h^0 \rightarrow t \tilde{t}_i \tilde{t}_j \]

  \[ \tilde{g} \rightarrow \tilde{t}_i \tilde{t}_j \]

  \[ \tilde{C}_k \rightarrow b \tilde{t}_i \tilde{t}_j \]

  etc.

  Large, diverse hierarchies of ratios of squared masses.

- **Method should be generic, reusable from start to finish.**

  Do calculations for scalars, fermions, vectors in a general field theory. Then apply to Higgs, squarks, sleptons, and quarks, gluino, charginos, neutralinos, . . .
To calculate physical masses

Evaluate self-energy = sum of 1-particle irreducible Feynman diagrams:

\[ \Pi(s) = \Pi^{(1)}(s) + \Pi^{(2)}(s) + \ldots \]

The complex pole mass

\[ s_{\text{pole}} = M^2 - i\Gamma M \]

is the solution of:

\[ s = m^2_{\text{tree}} + \Pi(s) \]

The pole mass is gauge invariant at each order in perturbation theory, can be related to kinematic masses as measured at colliders.

There are a finite number of two-loop, two-point Feynman diagrams. Why not just do them once, store the results, and get it over with?
Strategy:

- Reduce all self-energies in general theory to a few basis integrals
- Numerically evaluate basis integrals quickly and reliably for arbitrary values of masses.

Tarasov’s basis and recurrence relations:

\[ M \]

\[ S \]
\[ T \]
\[ U \]

Can always reduce 2-loop self-energies to a linear combination of these, with coefficients rational functions of:

- \( s = p^2 \) = external momentum invariant
- \( x, y, z, \ldots \) = internal propagator masses
To evaluate basis integrals:

Values at $s = 0$ are known analytically, in terms of logs, dilogs.

$$\frac{\partial}{\partial s} \text{(basis integral)} = \text{(another self-energy integral)}$$

$$= \text{(linear combination of basis integrals)}$$

So, we have a set of coupled, first-order, linear differential equations.

Consider the Master integral $M(x, y, z, u, v)$:

![Diagram of the Master integral with propagators](image)

and the basis integrals obtained from it by removing propagator(s):

$U(x, z, u, v), \ U(y, u, z, v), \ U(z, x, y, v), \ U(u, y, x, v), \ S(x, u, v), \ T(x, u, v), \ T(u, x, v), \ T(v, x, u), \ S(y, z, v), \ T(y, z, v), \ T(z, y, v), \ T(v, y, z)$

Call these 13 integrals $I_n, \ (n = 1, \ldots, 13)$. 
Differential equations method for basis integrals

\[ \frac{d}{ds} I_n = \sum_{m} K_{nm} I_m + C_n \]

Here $K_{nm}$ are rational functions of $s$ and $x, y, z \ldots$, and $C_n$ are one-loop integrals. These are obtained by using Tarasov’s recursion relations.

Solve for basis integrals $I_n$ using Runge-Kutta integration in the complex $s$-plane, starting from known values at $s = 0$.

Method implemented for $S, T, U$ type integrals by Caffo, Czyz, Laporta, Remiddi.

I have extended the method to also work for $M$. 
Advantages of the method:

- Basis integrals can be computed for any values of all masses and $s$, to arbitrary accuracy.

- All necessary basis integrals are obtained simultaneously in a single numerical computation.

- Branch cuts automatically dealt with correctly by choosing integration contour in upper-half complex $s$ plane.

- Simple checks on the numerical accuracy follow from changing choice of contour.

SPM, hep-ph/0307101
SPM and D.G. Robertson, C program, to appear.

Takes $\lesssim 0.05$ seconds on a modern workstation to compute each Master integral together with all subordinate integrals, for generic masses.
Application: 2-loop corrections to squark masses

- Dominant corrections are from SUSYQCD. Many of the integrals can be computed analytically in terms of polylogarithms.

- To a good approximation, can treat $W$ and $Z$ bosons as massless.

- For simplicity, I will show results assuming squarks are degenerate, no squark mixing, massless quarks. (Forthcoming paper treats general case.) Then all basis integrals are evaluated in terms of polylogarithms.

Counting all possible fermion mass insertions separately, but not diagrams related by reflections and permutations, there are 51 distinct topologies through two loop order...
SUSYQCD diagrams contributing to squark masses:
Checks on the calculation of squark pole masses:

- Independent of gauge-fixing parameter
  Individual diagrams depend on $\xi$; cancels in pole mass

- Pole mass is renormalization group invariant
  Checked analytically at 2-loop order; numerical check below

- Absence of divergent logs on shell
  Individual diagrams have $\log(1 - p^2/m^2)$, divergent as $p^2 \to m^2$; must and do cancel in pole mass

Example: In the special case of degenerate running masses $m_{\tilde{Q}} = m_{\tilde{g}} = Q$, the result simplifies:

$$M_{\tilde{Q},\text{pole}}^2 = m_{\tilde{Q}}^2 \left[ 1 + \frac{\alpha_s}{4\pi} \left( \frac{32}{3} \right) + \left( \frac{\alpha_s}{4\pi} \right)^2 \left\{ \frac{112}{3} + \frac{664\pi^2}{27} + \frac{32\pi^2\ln2}{9} - \frac{16\zeta(3)}{3} \right\} \right]$$

$$= m_{\tilde{Q}}^2 \left[ 1 + 0.849 \alpha_s + 1.89 \alpha_s^2 \right]$$

There are no large logs here (only one mass scale!), so this illustrates the intrinsic size of typical SUSYQCD one-loop ($\sim 4\%$) and two-loop ($\sim 1\%$) corrections to the squark masses.

Could reasonably guess three-loop corrections to be of order 0.3\%.
Renormalization scale ($Q$) dependence of calculated pole mass:

\[ \Delta M/M_{\text{squark}} \]

\[ Q/M_{\text{squark}} \]

\[ \alpha_s(M) = 0.095 \]
\[ M_{\text{gluino}} = M_{\text{squark}} \]

Squark mixing, quark masses, electroweak effects neglected; all squarks taken degenerate with each other and gluino at tree level.

Dashed lines are $\pm 2\%$ variation of $\alpha_s$. 

Remaining scale dependence (from 3 loops and beyond) is small.

2-loop correction is much larger than 1-loop scale dependence.
Dependence of squark mass correction on the gluino mass:

\[ \Delta M/M_{\text{squark}} \]

\[ \alpha_s(M_{\text{squark}}) = 0.095 \]

\[ Q = M_{\text{squark}} \]

A large part of the squark mass correction is due to the gluino mass.

In realistic models, effects due to variation in squark masses, top Yukawa effects, electroweak effects are significant, too. (Formulas, details in forthcoming paper.)
Application: Two-loop “momentum-dependent” corrections to Higgs mass in SUSY

All contributions to Higgs $\Pi(s)$ of order:

\[
\alpha_s y_t^2, \alpha_s y_b^2, \alpha_s y_t y_b, \\
\alpha_s g^2, \alpha_s g' g', \alpha_s g'^2 \\
y_t^4, y_b y_t, y_b^2 y_t, y_t^3 y_b, y_b^4, y_t^4, y_b^2 y_t^2
\]

are now included. (SPM hep-ph/0405022)

Previous results used the effective potential approximation, in which $\Pi^{(2)}(s)$ is approximated by $\Pi^{(2)}(0)$ when computing the pole mass. (That approximation relies on $m_{h0}^2 \ll m_t^2, m_{t}^2$.)

Most of the Feynman diagram topologies are the same as for squarks, evaluated in exactly same way; there are ten more.

The only 2-loop self-energy diagrams for scalars that remain are ones involving massive vector bosons.
Size of two-loop “momentum-dependent” effect is few hundred MeV.

A typical case:

![Graph showing $h^0$ pole mass vs. Renormalization scale $Q$]

However, remaining uncalculated 2-loop and leading 3-loop corrections will be necessary to compete with $\sim 200$ MeV uncertainty attainable at the LHC, or $\sim 50$ MeV at a future LC.
Summary

- 2-loop corrections to new particle masses are necessary

- Emphasis on flexible strategy; don’t want to bet all-or-nothing on the proposition that low-scale SUSY is correct!

- Squark mass corrections are significant at 2 loops (maybe even 3 loops)

- Gluino mass corrections are definitely significant at 2 loops (work in progress)

- neutralino, chargino, slepton, Higgs mass corrections can be done same way; calculations are “reusable”