Radiative Corrections to Neutrino-Nucleon Quasielastic Scattering

M. Fukugita (ICRR, Univ. of Tokyo) and T. Kubota (Osaka)

(dedicated to the memory of J. Kwieciński)


1. Introduction

Precise measurements in low-energy (1-10 MeV) weak processes

◊ KamLAND (Kamioka Liquid-scintillator AntiNeutrino Detector)

\[ \sigma(\bar{\nu}_e + p \rightarrow e^+ + n) \propto (f_V^2 + 3g_A^2) \quad f_V \equiv 1 \]

◊ SNO (Sudbury Neutrino Observatory)

\[ \sigma(\nu_e + d \rightarrow p + p + e^-) \propto g_A^2 \quad \text{CC} \]

\[ \sigma(\nu_x + d \rightarrow p + n + \nu_x) \propto g_A^2 \quad \text{NC} \quad \text{for} \quad ^3S \rightarrow ^1S \]

\[ [x = e, \mu, \tau] \]

◊ J-PARC  Asymmetry in neutron beta decay

\[ \text{Asymm} \propto \frac{g_A(f_V - g_A)}{(f_V^2 + 3g_A^2)} \quad f_V \equiv 1 \]

Knowledge of the cross sections and the decay rates with 1 per-cent accuracy required \[ \Rightarrow \]

QED and electro-weak radiative corrections
2. Radiative Corrections to $g_A = G_A/G_V$

A useful phenomenological parameter of the strong interactions for low-energy weak processes

$$\rightarrow g_A = 1.2670 \pm 0.0030 \quad \text{(PDG)}$$

$g_A - 1 = \text{effects largely of strong interactions}$

**Question**

What about the portion of QED/electroweak corrections to $g_A$?
Our Claim (on inner correction):

Electroweak corrections amount to replacing:

\[ f_V^2 \rightarrow f_V^2(1 + \delta^F_{\text{in}}) \quad \text{for CC} \]
\[ g_A^2 \rightarrow g_A^2(1 + \delta^{\text{GT}}_{\text{in}}) \quad \text{for CC} \]
\[ g_A^2 \rightarrow g_A^2(1 + \Delta^{\text{GT}}_{\text{in}}) \quad \text{for NC} \]

Note: \( \delta^F_{\text{in}} \neq \delta^{\text{GT}}_{\text{in}}, \delta^{\text{GT}}_{\text{in}} \neq \Delta^{\text{GT}}_{\text{in}} \) confirmed explicitly

The outer part \( \cdots \) studied to a considerable extent

The inner corrections to \( g_A \) \( \cdots \) not fully discussed

Points to be clarified:

\( \diamond \) **What is** \( G_A/G_V = 1.267 \),

\( g_A \) or \( g_A(1 + \delta^{\text{GT}}_{\text{in}} - \delta^F_{\text{in}})^{1/2} \) ? \( (\delta^{\text{GT}}_{\text{in}} \neq \delta^F_{\text{in}}) \)

\( \diamond \) \( g_A \) in beta-asymmetry = \( g_A \) in neutron decay rate

\( \diamond \) \( g_A \) in CC \( \neq g_A \) in NC
3. Calculational Algorithm

(i) QED corrections: \( 0 < |k|^2 < M^2 \)

\(M^2\): cutoff introduced by hand \( m_p^2 \ll M^2 \ll m_W^2 \)

Four-Fermi interactions of nucleons

\[ \longrightarrow \text{IR-finite, but UV-divergent} \]

\[
\mathcal{M}^{(0)} \equiv \frac{G_V}{\sqrt{2}} [\bar{u}_e(\ell)\gamma^\lambda(1 - \gamma^5)\nu_\nu(p_\nu)] [\bar{u}_p(p_2)W_\lambda(p_2, p_1)u_n(p_1)]
\]

\[
W_\lambda(p_2, p_1) = \gamma_\lambda(f_V - g_A\gamma^5), \quad (f_V \equiv 1)
\]

(ii) Electroweak Corrections: \( M^2 < |k|^2 < +\infty \)

Weinberg-Salam theory of quarks \( \longrightarrow \text{UV-convergent} \)

Key Questions

(1) How to connect the two regions (i) and (ii) smoothly

(2) How to handle the effects due to strong int.
図 1: QED corrections to $\bar{\nu}_e + p \rightarrow e^+ + n$

図 2: Electroweak corrections to $\bar{\nu}_e + p \rightarrow e^+ + n$
4. UV-divergence in four-Fermi Theory

(i) To be eliminated by the Weinberg-Salam theory

\[
\log \left( \frac{M^2}{m_p^2} \right) + \log \left( \frac{m_W^2}{M^2} \right) = \log \left( \frac{m_W^2}{m_p^2} \right)
\]

\[\uparrow \quad \uparrow\]

QED Electroweak

Are coefficients not affected by strong interactions?

(ii) To be eliminated by considering extended nucleon

Form factors in nucleon’s e.m. and weak vertices render the Feynman integrals finite.

- Classification of log-div. into (i) and (ii) is possible via CVC, PCAC and Current Algebra techniques even for Gamow-Teller part. (\(\leftarrow\) M. Fukugita and T.K. (’04))

- Universality of log-coeff. can be proved.
UV and IR divergences

<table>
<thead>
<tr>
<th></th>
<th>QED</th>
<th>electroweak</th>
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<tbody>
<tr>
<td></td>
<td>vertex</td>
<td>self-energy</td>
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</tr>
<tr>
<td>v2</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td>v3</td>
<td>yes</td>
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<tr>
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<tr>
<td>IR</td>
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v1 + self-energy + brems. → outer corrections

v2 + v3 + electroweak → inner corrections

◊ The log$M^2$ terms in v2 are connected smoothly with those in Weinberg-Salam.

◊ The log$M^2$ terms in v3 are softened by form factors.
5. Summary of Our Calculations

\[
\frac{d\sigma(\bar{\nu}_e + p \rightarrow e^+ + n)}{d(\cos\theta)} = \frac{G_V^2}{2\pi} E^2 \beta \{ A(\beta) + B(\beta) \beta \cos\theta \},
\]

\[G_V = G_F \cos\theta_C, \quad \theta_C = \text{Cabibbo angle}\]

\[E, \beta = \text{energy and velocity of the final positron}\]

\[\theta = \text{scattering angle of the positron}\]

At the tree level

\[A(\beta) = A_0 \equiv f_V^2 \langle 1 \rangle^2 + g_A^2 \langle \sigma \rangle^2,\]

\[B(\beta) = B_0 \equiv f_V^2 \langle 1 \rangle^2 - \frac{1}{3} g_A^2 \langle \sigma \rangle^2\]

At the one-loop level

\[A(\beta) = \left\{ 1 + \delta_{\text{out}}(E) \right\} \left( \bar{f}_V^2 \langle 1 \rangle^2 + \bar{g}_A^2 \langle \sigma \rangle^2 \right)\]

\[B(\beta) = \left\{ 1 + \tilde{\delta}_{\text{out}}(E) \right\} \left( \bar{f}_V^2 \langle 1 \rangle^2 - \frac{1}{3} \bar{g}_A^2 \langle \sigma \rangle^2 \right)\]

\[\bar{f}_V^2 = f_V^2 \left( 1 + \delta_m^F \right), \quad \bar{g}_A^2 = g_A^2 \left( 1 + \delta_m^{GT} \right)\]
<table>
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<tr>
<th>( \delta_{\text{out}}(E) )</th>
<th>Vogel (1984) and Fayans (1985)</th>
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<tbody>
<tr>
<td>( \tilde{\delta}_{\text{out}}(E) )</td>
<td>Fukugita and T.K. ('04)</td>
</tr>
<tr>
<td>( \delta_{\text{in}}^F )</td>
<td>Marciano-Sirlin (1986), Towner (1992)</td>
</tr>
<tr>
<td>( \delta_{\text{in}}^{GT} )</td>
<td>M. Fukugita and T.K. ('04)</td>
</tr>
</tbody>
</table>

**Our Values** \( \delta_{\text{in}}^F = 0.02370 \pm 0.0008, \)

\[
\delta_{\text{in}}^{GT} = 0.02616 \pm 0.0008
\]

\( \delta_{\text{in}}^F \) and \( \delta_{\text{in}}^{GT} \) agree with those in:

(i) Neutron beta decay rate

(ii) Asymmetry in polarised neutron beta decay

(iii) \( \nu_e + d \rightarrow p + p + e^- \) (SNO CC).
\[
\delta_{\text{in}}^F = \frac{e^2}{8\pi^2} \left\{ \frac{3}{2} \log \left( \frac{m_Z^2}{m_p^2} \right) + 3\bar{Q}\log \left( \frac{m_Z^2}{M^2} \right) + C^F \right\} \\
C^F = 1.751 + 0.409 = 2.160
\]

\[
\delta_{\text{in}}^{\text{GT}} = \frac{e^2}{8\pi^2} \left\{ \frac{3}{2} \log \left( \frac{m_Z^2}{m_p^2} \right) + 1 + 3\bar{Q}\log \left( \frac{m_Z^2}{M^2} \right) + C^{\text{GT}} \right\} \\
C^{\text{GT}} = 0.727 + 2.554 = 3.281
\]

\[
\delta_{\text{out}}(E) = \frac{e^2}{8\pi^2} \left[ \frac{23}{4} + \frac{3}{2} \log \left( \frac{m_p^2}{m_e^2} \right) + \frac{8}{\beta} L \left( \frac{2\beta}{1+\beta} \right) + 4\log \left( \frac{4\beta^2}{1-\beta^2} \right) \left( \frac{1}{\beta} \tanh^{-1}\beta - 1 \right) - \frac{8}{\beta} (\tanh^{-1}\beta)^2 + \left( \frac{7}{2\beta} + \frac{3\beta}{2} \right) \tanh^{-1}\beta \right]
\]

\[
\tilde{\delta}_{\text{out}}(E) = \frac{e^2}{8\pi^2} \left[ \frac{3}{4} + \frac{3}{2} \log \left( \frac{m_p^2}{m_e^2} \right) + \frac{8}{\beta} L \left( 1 - \sqrt{\frac{1-\beta}{1+\beta}} \right) + \frac{4}{\beta^2} - 4\sqrt{1-\beta^2} \right] \\
+ 4 \left( 1 - \frac{1}{\beta} \tanh^{-1}\beta \right) \log \left( \frac{1}{2} \left( 1 + \frac{1}{\beta} \right) \sqrt{\frac{1+\beta + \sqrt{1-\beta}}{1+\beta - \sqrt{1-\beta}}} \right) + \left( \frac{1}{\beta} - 4 \right) \tanh^{-1}\beta + \left( \frac{2}{\beta} - \frac{3}{2} - \frac{1}{2\beta^2} \right) (\tanh^{-1}\beta)^2 \right]
\]
$\delta_{\text{out}}(E)$ (solid line) and $\tilde{\delta}_{\text{out}}(E)$ (dotted line)
6. Neutral Current \((\nu_x + d \rightarrow \nu_x + n + p)\)

No QED corrections (no outer corrections)

Electroweak radiative corrections
Effective four-fermi interactions of quarks

\[
\mathcal{M}_{\text{eff}} = \frac{G_{\mu}}{\sqrt{2}} \frac{im_Z^2}{q^2 - m_Z^2} \rho_{\text{NC}}^{(\nu;h)} \bar{u}_\nu \gamma^\mu (1 - \gamma^5) u_\nu \\
\times \left\{ \bar{\psi} I_3 \gamma_\mu (1 - \gamma^5) \psi - 2 K^{(\nu;h)}(q^2) \sin^2 \theta_W \bar{\psi} \gamma_\mu Q \psi \right\}
\]

Marciano-Sirlin (1981)

No worry about \(\log M^2\) cancellation
\[
\rho_{NC}^{(\nu;h)} = \left( 1 + \Delta_{\text{in}}^{\text{GT}} / 2 \right)
\]
\[
= \begin{cases} 
1.00955 & \text{for } m_H = 1.5m_Z \\
1.00862 & \text{for } m_H = 5m_Z 
\end{cases}
\]

\(K^{(\nu;h)}\): irrelevant to Gamow-Teller transition

SNO NC process •••••• pure Gamow-Teller

\((3S \rightarrow 1S)\)

\[
\mathcal{M}_{\text{eff}} = i \frac{G_F}{\sqrt{2}} g_A \rho_{NC}^{(\nu;h)} \left[ \bar{u}_\nu \gamma^\mu (1 - \gamma^5) u_\nu \right] [\bar{\psi}_N I_3 \gamma_\mu \gamma^5 \psi_N]
\]

g_A has been replaced by \(g_A\rho_{NC}^{(\nu;h)} = g_A(1 + \Delta_{\text{in}}^{\text{GT}} / 2)\).
7. Summary

We have computed full radiative corrections to:

\[
\begin{align*}
\text{neutron-decay rate} & \quad \Gamma(n \rightarrow p + e^- + \bar{\nu}_e) \\
\text{J-PARC} & \quad \text{Asymm}(n \rightarrow p + e^- + \bar{\nu}_e), \\
\text{KamLAND} & \quad d\sigma(\bar{\nu}_e + p \rightarrow e^+ + n), \\
\text{SNO CC} & \quad d\sigma(\nu_e + d \rightarrow p + p + e^-), \\
\text{SNO NC} & \quad d\sigma(\nu_x + d \rightarrow n + p + \nu_x),
\end{align*}
\]

and have confirmed that the low-energy parameter $g_A^2$ is replaced 
universally by $g_A^2(1 + \delta_{\text{in}}^{\text{GT}})$ for CC and by $g_A^2(1 + \Delta_{\text{in}}^{\text{GT}})$ for NC.

We have, in particular, found $\delta_{\text{in}}^F \neq \delta_{\text{in}}^{\text{GT}}, \delta_{\text{in}}^{\text{GT}} \neq \Delta_{\text{in}}^{\text{GT}}$ and

\[
G_A/G_V = \frac{\bar{g}_A}{f_V} = g_A(1 + \delta_{\text{in}}^{\text{GT}} - \delta_{\text{in}}^F)^{1/2} \neq g_A.
\]

In the future precise data analysis in $\nu$-reactions, we have to distinguish $g_A^2, g_A^2(1 + \delta_{\text{in}}^{\text{GT}})$, and $g_A^2(1 + \Delta_{\text{in}}^{\text{GT}})$. 