How Stueckelberg extends the (Supersymmetric) Standard Model

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Stueckelberg mechanism for gauge boson masses

- The naive Lagrangian of a massive abelian vector boson:
  \[
  \mathcal{L}_{\text{St}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{m^2}{2} A_\mu A^\mu
  \]
  Split off longitudinal mode of \( A_\mu \) via
  \[
  A_\mu \rightarrow A_\mu + \frac{1}{m} \partial_\mu \sigma
  \]
  and define gauge transformation
  \[
  \delta A_\mu = \partial_\mu \epsilon \ , \ \delta \sigma = -m \epsilon
  \]
  Gauge invariant renormalizable mass term without Higgs!

- Unitary gauge by adding
  \[
  \mathcal{L}_{\text{gf}} = -\frac{1}{2\xi} (\partial_\mu A^\mu + \xi m \sigma)^2
  \]
  and \( A_\mu \) and \( \sigma \) decouple:
  \[
  \mathcal{L}_{\text{St}} + \mathcal{L}_{\text{gf}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{m^2}{2} A_\mu A^\mu - \frac{1}{2\xi} (\partial_\mu A^\mu)^2
  \]
  \[
  -\frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \xi \frac{m^2}{2} \sigma^2
  \]

- Note:
  1. Vector \( A_\mu \) eats (real) scalar \( \sigma \) with nothing left.
  2. Constant PQ-shift symmetry \( \delta \sigma = c \): axionic pseudoscalar.
  3. Gauge invar. Lagr. exists only for abelian gauge symmetry.
Stueckelberg in string theory/SUGRA

- $10d \mathcal{N} = 1$ SUGRA coupled to SYM: 2-form $B_{IJ}$ modified
  \[
  (\partial_{[I} B_{JK]})^2 \longrightarrow (\partial_{[I} B_{JK]} + \frac{\kappa_{10}}{g_{YM}} A_{[I} F_{JK]} + \cdots)^2
  \]
  Reduction with internal gauge field $\langle F_{ij} \rangle \neq 0:
  \[
  (\partial_{\mu} B_{ij} + \frac{\kappa_{10}}{g_{YM}} A_{\mu} F_{ij} + \cdots)^2 \sim (\partial_{\mu} \sigma + m A_{\mu})^2
  \]
  with $\sigma \sim B_{ij}$ and $m \sim \langle F_{ij} \rangle$: "Topological mass"

- Well known from Green-Schwarz mechanism:
  \[
  mA^{\mu} \partial_{\mu} \sigma + c \sigma F_{\mu\nu} \tilde{F}^{\mu\nu}
  \]
  contribution to chiral gauge anomalies (in 4$d$)

\[A_{\mu} \sigma + A_{\mu} = 0\]

Contribution to anomalous triangle diagram $= m \cdot c$.
But mass parameter $= m$.

- Anomalous $U(1)$ always gets massive: $m \cdot c \neq 0$.
  Non-anomalous $U(1)$ can still get massive: $m \neq 0, c = 0$.
- For Stueckelberg models assume $m \neq 0, c = 0$.
- Mass scale $\sim$ string or compactification scale.
Stueckelberg in D-brane models

- Intersecting brane world (four stack) SM:

\[
\text{IBW on } T^6 = T_1^2 \times T_2^2 \times T_3^6
\]

Gauge group (at best):

\[
U(3) \times U(2) \times U(1)^2 \xrightarrow{\text{Stueckelberg}} SU(3) \times SU(2)_L \times U(1)_Y
\]

Stueckelberg masses for extra \( U(1) \)'s:

\[
m_{ij}^2 \sim \tan(\varphi_i) \tan(\varphi_j) \quad \text{for } U(1)_iU(1)_j
\]

- More general IBW configurations on Calabi-Yau 3-folds
**Simplest Stueckelberg extension of the SM**

- Add one $U(1)_X +$ axion with Stueckelberg couplings to SM:

\[
\begin{align*}
\text{Higgs } \Phi & : & SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_X \\
\text{Stueckelberg } \sigma & : & SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_X
\end{align*}
\]

Extra degrees of freedom $C_\mu + \sigma \longrightarrow$ massive vector $Z'$

- Lagrangian of minimal Stueckelberg extension of the SM

\[
\mathcal{L}_{\text{SM}} = -\frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + g_2 A^a_\mu J^a_2 + g_Y B_\mu J^\mu_Y - D_\mu \Phi^\dagger D^\mu \Phi - V(\Phi^\dagger \Phi) + \cdots
\]

\[
\mathcal{L}_{\text{St}} = -\frac{1}{4} C_{\mu\nu} C^{\mu\nu} + g_X C_\mu J^\mu_X - \frac{1}{2} (\partial_\mu \sigma + M_1 C_\mu + M_2 B_\mu)^2
\]

Gauge invariance independent for $U(1)_Y \times U(1)_X$:

- **Hypercharge**: $\delta_Y B_\mu = \partial_\mu \epsilon_Y$, $\delta_Y \sigma = -M_2 \epsilon_Y$
- **$U(1)_X$**: $\delta_X C_\mu = \partial_\mu \epsilon_X$, $\delta_X \sigma = -M_1 \epsilon_X$

Further add gauge fixing $\mathcal{L}_{\text{gf}}$ as before.

- Assumptions for simplest scenario:
  1. SM matter neutral under $U(1)_X$.
  2. Hidden sector neutral under SM gauge group.
  3. No spontaneous breaking of $U(1)_X$ in hidden sector.
Stueckelberg effects in the SM

- Only effect is on vector boson mass matrix (basis \((C_\mu, B_\mu, A^3_\mu)\)):

\[
\begin{bmatrix}
M_1^2 & M_1 M_2 & 0 \\
M_1 M_2 & M_2^2 + \frac{1}{4}g_Y^2 v^2 & -\frac{1}{4}g_Y g_2 v^2 \\
0 & -\frac{1}{4}g_Y g_2 v^2 & \frac{1}{4}g_2^2 v^2
\end{bmatrix}
\]

Massless photon \(M^2_\gamma = 0\), two massive eigenvalues

\[
M^2_Z = \frac{v^2}{4} (g_2^2 + g_Y^2) + O(\delta), \quad M^2_{Z'} = M^2 + O(\delta)
\]

Two new parameters: mass scale + small coupling factor

\[
M^2 = M^2_1 + M^2_2 > [150 \text{ GeV}]^2, \quad \delta = \frac{M_2}{M_1} < 0.01
\]

Cplgs suppr. by \(\delta\): EW fits intact even with low mass \(M\).

- Comparison to \(U(1)'\) models with Higgs:

Suppression factor: \(\frac{M^2_Z}{M^2_{Z'}} \sim 0.01 \rightarrow \delta\)

Stueckelberg: No other fields than \(Z'\) left.

- Planck-scale effects at low energies?

\(M_1, M_2 \rightarrow \infty, \quad \delta = \text{finite}\)

\(Z'\) very heavy, but small effects remain.

[Hewett, Rizzo; Cvetic, Langacker; Leike; and many others]
Stueckelberg effects in the SM

• Diagonalize vector mass matrix by

\[ \mathcal{O}^T \cdot M_{ab}^2 \cdot \mathcal{O} = \text{diag}(M_{Z'}^2, M_Z^2, 0) , \quad \mathcal{O} = \mathcal{O}(\theta, \phi, \psi) \]

with

\[ \tan(\phi) = \delta , \quad \tan(\theta) = \frac{g_Y}{g_2} \cos(\phi) , \quad \tan(\psi) = \frac{g_2(1 - M^2/M_{Z'}^2)}{g_Y \sin(\phi) \cos(\theta)} \]

Bounds: \( \phi, \psi < 1^0 \), \( \theta \rightarrow \theta_W \)

• Couplings to matter fields

\[ \mathcal{L}_{\text{int}} = g_2 A^a_{\mu} J_2^{a \mu} + g_Y B_{\mu} J_Y^\mu + g_X C_{\mu} J_X^\mu \]

for the photon find

\[ e A^\gamma_{\mu} J^\mu_{\text{em}} = \frac{g_2 g_Y \cos(\phi)}{\sqrt{g_2^2 + g_Y^2 \cos^2(\phi)}} \underbrace{A^\gamma_{\mu} \left( J_Y^\mu + J_2^{3 \mu} - \frac{g_X}{g_Y} \tan(\phi) J_X^\mu \right)}_{\text{electric charge}} \]

Electric charge of hidden sector irrational (and small).

• Decay widths and branching ratios of \( Z' \):

\[ \Gamma(Z' \rightarrow f \bar{f}) \sim 10 \text{ MeV} \]

\( Z' \) very sharp peak in \( e^+e^- \) (among other signatures).
Stueckelberg extension of the MSSM: StMSSM

• Supersymmetrize Stueckelberg Lagrangian

\[ \mathcal{L}_{\text{St}} = \int d^2 \theta d^2 \bar{\theta} \left( M_1 C + M_2 B + S + \bar{S} \right)^2 \]

with \( S \) Stueckelberg chiral multiplet, \( B, C \) vector multiplets:

\[ S = (\chi, \rho + i\sigma, F) \]

\[ B = (B_\mu, \lambda_B, D_B) \]

\[ C = (C_\mu, \lambda_C, D_C) \]

in components:

\[ \mathcal{L}_{\text{St}} = -\frac{1}{2}(M_1 C_\mu + M_2 B_\mu + \partial_\mu \sigma)^2 - \frac{1}{2}(\partial_\mu \rho)^2 - \frac{i}{2}(\chi \sigma^\mu \partial_\mu \bar{\chi} - (\partial_\mu \chi) \sigma^\mu \bar{\chi}) + 2|F|^2 + \rho(M_1 D_C + M_2 D_B) + [\bar{\chi}(M_1 \lambda_C + M_2 \lambda_B) + \text{h.c.}] \]

• Eliminate auxiliary fields \( F, D_B, D_C \): corrected D-terms.

• Add soft breaking terms

\[ \mathcal{L}_{\text{soft}} = -\frac{1}{2} \tilde{m}_\rho^2 \rho^2 - \frac{1}{2} \tilde{m}_1 \lambda_B \lambda_B - \frac{1}{2} \tilde{m}_C \lambda_C \lambda_C - \frac{1}{2} m_1^2 |h_1|^2 - \frac{1}{2} m_2^2 |h_2|^2 - m_3^2 (h_1 \cdot h_2 + \text{h.c.}) \]

• Compare to Higgs: “Stueckelino” \( \chi \) neutral

No cplg. \( g_Y B_\mu \chi \sigma^\mu \bar{\chi} \), \( g_X C_\mu \chi \sigma^\mu \bar{\chi} \)

No anomalous contribution \( \Rightarrow \) no second multiplet needed.
Stueckelberg effects in the StMSSM

- Scalar potential with \( \rho \) plus Higgs \( h_1, h_2 \):

\[
V(h_1, h_2, \rho) = \frac{1}{2}(M_1^2 + M_2^2 + \tilde{m}_\rho^2)\rho^2 + V_D^{\text{MSSM}}(h_1, h_2)
+ \frac{1}{2}(m_1^2 - \rho g_Y M_2)|h_1|^2 + \frac{1}{2}(m_2^2 + \rho g_Y M_2)|h_2|^2 + m_3^2(h_1 \cdot h_2 + \text{h.c.})
\]

new scalar \( \rho \) shifts Higgs mass terms through vev: \( \rho \rightarrow v_\rho + \rho \).

Modification of EW constraint negligible:

\[
\frac{1}{2} M_0^2 = \frac{m_1^2 - m_2^2 \tan^2(\beta)}{\tan^2(\beta) - 1} + \frac{g_Y M_2 v_\rho}{\cos(2\beta)}, \quad |g_Y M_2 v_\rho| < 10^{-4} M_Z^2
\]

- Scalar mass matrix (3 CP-even states: \( h_1, h_2, \rho \)):

\[
\begin{bmatrix}
M_0^2 c_\beta^2 + m_A^2 s_\beta^2 & -(M_0^2 + m_A^2) s_\beta c_\beta & -t_\theta c_\beta M_W M_2 \\
-(M_0^2 + m_A^2) s_\beta c_\beta & M_0^2 s_\beta^2 + m_A^2 c_\beta^2 & t_\theta s_\beta M_W M_2 \\
-t_\theta c_\beta M_W M_2 & t_\theta s_\beta M_W M_2 & M^2 + \tilde{m}_\rho^2
\end{bmatrix}
\]

Third resonance (mass eigenstate) in \( J = 0^+ \) channel:

\[
\Gamma(\rho_S \rightarrow t\bar{t}) \sim \text{MeV}
\]

again very sharp.
Stueckelberg effects in the StMSSM

- Neutralino mass matrix now with $\chi$, $\lambda_C$ plus usual 4: $6 \times 6$

$$
\begin{bmatrix}
0 & M_1 & M_2 & 0 & 0 & 0 \\
M_1 & \tilde{m}_X & 0 & 0 & 0 & 0 \\
M_2 & 0 & \tilde{m}_1 & 0 & -c_1M_0 & c_2M_0 \\
0 & 0 & 0 & \tilde{m}_2 & c_3M_0 & -c_4M_0 \\
0 & 0 & -c_1M_0 & c_3M_0 & 0 & -\mu \\
0 & 0 & c_2M_0 & -c_4M_0 & -\mu & 0 \\
\end{bmatrix}
$$

with $c_1 = c_\beta s_\theta$, $c_2 = s_\beta s_\theta$, $c_3 = c_\beta c_\theta$, $c_4 = s_\beta s_\theta$.

- Split eigenstates into $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0}$ and

$$
m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_6^0} = \sqrt{M_1^2 + \frac{1}{4}\tilde{m}_X^2 \pm \frac{1}{2}\tilde{m}_X + \mathcal{O}(\delta)}, \quad m_{\tilde{\chi}_5^0} \geq m_{\tilde{\chi}_6^0}
$$

Interesting regime: $m_{\tilde{\chi}_6^0} < m_{\tilde{\chi}_1^0} \Rightarrow \tilde{\chi}_6^0$ LSP of StMSSM!

- Then see decay cascades:

$$
\tilde{\chi}_1^0 \rightarrow l_i\tilde{l}_i\tilde{\chi}_1^0, \quad q_j\tilde{q}_j\tilde{\chi}_1^0, \quad Z\tilde{\chi}_1^0
$$

of sleptons

$$
\tilde{l}^- \rightarrow \tilde{l}^- + \tilde{\chi}_1^0 \rightarrow \tilde{l}^- + \left\{l_i^--l_i^+ + \{\tilde{\chi}_1^0\}ight\}
$$

charginos

$$
\tilde{\chi}_1^- \rightarrow \tilde{l}^- + \tilde{\chi}_1^0 + \tilde{\nu} \rightarrow \tilde{l}^- + \left\{l_i^--l_i^+ + \{\tilde{\chi}_1^0 + \tilde{\nu}\}ight\}
$$

and so on.
Summary

1. The Stueckelberg mechanism is a gauge invariant, renormalizable way to generate gauge boson masses for abelian vector fields.

2. It naturally appears in many models that descend from string theory and higher-dimensional SUGRA, with “topological” mass terms, related to GS anomaly cancellation.

3. It is very “economic” and distinct from Higgs models with extra $U(1)'$ gauge factors, already at the level of the degrees of freedom.

4. In the SM, it only affects the vector boson mass matrix with an extra $Z'$, and induces small exotic couplings of the photon and $Z$.

5. In the MSSM, it introduces one extra scalar $\rho$, and two new neutralinos, besides $Z'$. In a wide range of parameter space, one of the new neutralinos can be the new LSP, and therefore affect SUSY signatures significantly.